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The ground- and first-excited states of magnetopolarons in two-dimensional quantum dots for all coupling strengths

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Abstract. The ground- and first-excited state energies of a magnetopolaron in a two dimensional parabolic quantum dot are studied within a variational calculation for all coupling strength. The Lee-Low-Pines-Huybrecht variational technique that is developed previously for all coupling strength has been extented for polarons in a magnetic field. The dependence of the polaronic correction on the magnetic field and the confinement length is investigated. The polarization potential and the renormalized cyclotron masses as a function of electron-phonon coupling strength and the strength of both confinement potential and magnetic field are also studied within the same approach.

PACS. 73.63.Kv Quantum dots – 71.38.Fp Large or Fröhlich polarons – 63.20.Kr Phonon-electron and phonon-phonon interactions

1 Introduction

Recent technological advances in the fabrication of nanostructures have created low-dimensional semiconductors, of which quantum dots (QD) have drawn more attraction as being confined in all the three spatial directions. This confinement is very important since it brings in quantum effects when the electron wavelength is of the same order as the confinement length. Recently, a large number of theoretical and experimental investigations have gone into understanding and exploring various properties of these systems [1].

Electron-phonon interaction, which plays an important role in electronic and optical properties of bulk materials, will have pronounced effect in low-dimensional systems, in particular in QDs. Polaronic effects in QDs are studied of every aspects: the ground-state energies are calculated for a QD with spherical [2] and parabolic [3] potentials. The presence of a magnetic field makes the polaron problem in QDs more interesting, since its existence means an additional confinement. Recently, a number of authors have extensively studied magnetopolarons in various type of approximations [4], and an recent review of the subject has been given by Devreese [5]. It should be mentioned that the rigorous all-coupling method in the polaron theory is based on the Feynman path integral method, where the Jensen-Feynman inequality reveals some problems for nonzero magnetic field. This problem, however, has been solved by a recent work [6].

In the weak coupling case the states are extended and perturbation theory can be used, while in the other strong coupling limit, which corresponds to localized states, adiabatic methods are employed. Between them, variational techniques are adopted, where certain canonical transformations called Lee-Low-Pines (LLP) are performed and each of which has its own variational parameters. One of these transformations is modified by Huybrechts [7] to extend this approach to all coupling strength. This LLPH method has been successfully employed in calculation of the polaronic corrections to the ground- and first-excited states in QD by Mukhopadhyay and Chatterjee [8] (MC). The polaronic effects in QDs have also been examined by the Feynman-Haken path integral method [9] in the absence of a magnetic field.

The polaronic effects in low dimensional systems have been subjected to experimental investigations by means of optical properties and cyclotron resonance studies. For examples, experiments made recently in QD's are far-infrared optical spectroscopic measurements [10], the multiphonon photoluminescence [11] and Raman [12] spectra. Cyclotron resonance absorption and emission have been performed in GaAs [13]. A recent interesting work [14] investigates resonant electron-phonon interaction in a semiconductor QD, which predicts pinning of the excited energy levels to the ground state level plus one optical phonon, whose effect is to be observable through optical spectroscopic measurements.

In this paper we shall consider polaronic corrections in a QD embedded in two dimension, and in magnetic fields. We shall employ the LLPH method to obtain the ground- and first relaxed excited state energies of a magnetopolaron embedded in 2D QD and related cyclotron masses. We also consider the polarization potential in the presence of magnetic field, which is the direct measure of the

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induced charge density and was first examined by Peeters *et al.* [15] for magnetopolarons in bulk systems. Although the Feynman-Haken path integral approach to the problem yields more accurate results than the LLPH method, our approach provides results not only for ground state energy but also for the relaxed excited state energies and improves the result for the ground state energy of reference [8]. The layout of the present work is as follows: in Section 2, the formulation of the problem is presented in the framework of the LLPH method. The limiting cases, that is, extended and localized states are discussed in Section 3. The ground- and first-excited states are examined numerically in Sections 4 and 5, respectively, and the paper ends with a brief conclusion.

2 Theory

An electron interacting with LO phonons in an isotropic harmonic potential and a magnetic field along the z-direction is described by the Fröhlich Hamiltonian

$$H=\frac{1}{2\mu}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2+\frac{1}{2}\mu\omega_\bullet^2\mathbf{r}^2+\sum_{\mathbf{q}}\hbar\omega_0 b_{\mathbf{q}}^\dagger b_{\mathbf{q}} +\sum_{\mathbf{q}}\left(V_{\mathbf{q}}b_{\mathbf{q}}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}}+\mathrm{h.c.}\right) \quad (1)$$

where $\mathbf{p}=\hbar\mathbf{k}$ is the electron momentum with a band mass μ and $b_{\mathbf{q}}^\dagger(b_{\mathbf{q}})$ is the creation (annihilation) operator of an optical phonon with a wave vector $\mathbf{q}$. The second term in equation (1) is a parabolic potential which confines the electron to zero dimension and so produces the QD with confinement frequency $\omega_\bullet$. In the last term representing the electron-phonon interaction, $V_{\mathbf{q}}$ is defined by [16]

$$|V_{\mathbf{q}}|^2=\left(\frac{2\pi\alpha}{V}\right)(\hbar\omega_0)^2\frac{r_0}{q} \quad (2)$$

where $r_0=(\hbar/2\mu\omega_0)^{1/2}$ is the polaron radius.

In order to solve equation (1) within the scheme of our variational approach we use a variational state vector $|\Psi\rangle=U_1U_2|0\rangle_{\mathbf{ph}}\otimes|m\rangle$ which is the direct product of wave function for the phonon part generated from the vacuum by successive canonical transformations

$$U_1=\exp\left[-\mathrm{i}\lambda\mathbf{r}\cdot\sum_{\mathbf{q}}\mathbf{q}\,b_{\mathbf{q}}^\dagger b_{\mathbf{q}}\right] \quad (3)$$

and

$$U_2=\exp\left[\sum_{\mathbf{q}}\left(b_{\mathbf{q}}^\dagger f_{\mathbf{q}}-b_{\mathbf{q}}f_{\mathbf{q}}^*\right)\right] \quad (4)$$

by the wave function for the electronic part whose coordinate representation is given by

$$\langle\mathbf{r}\mid\mathbf{m}\rangle=\frac{\eta^{|m|+1}}{\sqrt{\pi\,|m|!}}\mathrm{e}^{-\eta^2\mathbf{r}^2/2}\,|r|^m\,\mathrm{e}^{\mathrm{i}m\varphi} \quad (5)$$

where $f_{\mathbf{q}}$ is a variational function, and λ and η are variational parameters. U_2 generates coherent phonon states acting as $U_2^{-1}b_{\mathbf{q}}U_2=F_{\mathbf{q}}=b_{\mathbf{q}}+f_{\mathbf{q}}$, while U_1transforms the exponential term in equation (1), acting as $U_1^{-1}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}}U_1=\mathrm{e}^{\mathrm{i}(1-\lambda)\mathbf{q}\cdot\mathbf{r}}$ and eliminates the electron coordinates from the Hamiltonian in the case of $\lambda=1$ (extended state limit), and has no effect on electronic coordinates in the case of $\lambda=0$ (localized state limit).

After these two successive transformations the Hamiltonian becomes

$$\mathcal{H}=\frac{1}{2\mu}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2+\frac{1}{2}\mu\omega_\bullet^2\mathbf{r}_\perp^2+\sum_{\mathbf{q}}C_q(\mathbf{r})F_{\mathbf{q}}^\dagger F_{\mathbf{q}} +\sum_{\mathbf{q}}\left(V_{\mathbf{q}}F_{\mathbf{q}}\mathrm{e}^{\mathrm{i}(1-\lambda)\mathbf{q}\cdot\mathbf{r}}+\mathrm{h.c.}\right) +\sum_{\mathbf{q}}\sum_{\mathbf{k}}\frac{\lambda^2\hbar^2}{2\mu}\mathbf{q}\cdot\mathbf{k}\,F_{\mathbf{q}}^\dagger F_{\mathbf{k}}^\dagger F_{\mathbf{q}}F_{\mathbf{k}} \quad (6)$$

where $\mathcal{H}=U_2^{-1}U_1^{-1}HU_1U_2$ and

$$C_q(\mathbf{r})=\hbar\omega_0-\frac{\lambda\hbar}{\mu}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)\cdot\mathbf{q}+\frac{\lambda^2\hbar^2q^2}{2\mu}\cdot \quad (7)$$

The wave function in equation (5) can be deduced from a more general trial wave function used for the ground- and excited states in reference [17]. Hence, the expectation value of the Hamiltonian becomes

$$\langle\Psi|\,H\,|\Psi\rangle=\left\langle\frac{1}{2\mu}\left(\mathbf{p}+\frac{e}{c}\mathbf{A}\right)^2\right\rangle_m+\left\langle\frac{1}{2}\mu\omega_\bullet^2\mathbf{r}_\perp^2\right\rangle_m +\sum_{\mathbf{q}}\overline{C}_q\,|f_{\mathbf{q}}|^2+\sum_{\mathbf{q}}\left[V_{\mathbf{q}}f_{\mathbf{q}}\sigma_m(\mathbf{q})+V_{\mathbf{q}}^*f_{\mathbf{q}}^*\sigma_m^*(\mathbf{q})\right] \quad (8)$$

where $\sigma_m(\mathbf{q})=\left\langle\mathrm{e}^{\mathrm{i}(1-\lambda)\mathbf{q}\cdot\mathbf{r}}\right\rangle_m$. In obtaining equation (8), the diagonal part of the transformed Hamiltonian has been taken into account, and it is assumed that $\sum_{\mathbf{q}}\mathbf{q}\,|f_{\mathbf{q}}|^2=0$, due to the symmetry of QD. Therefore $C_q(\mathbf{r})$ reduces $\overline{C}_q(\mathbf{r})=\hbar\omega_0+\frac{\lambda^2\hbar^2}{2\mu}q^2$. Furthermore, in equation (8), if the symmetrical Coulomb gauge $\mathbf{A}=B(-y,x,0)/2$ is chosen for the vector potential and minimization of the resultant equation with respect to $f_{\mathbf{q}}$ is performed, then we obtain

$$f_{\mathbf{q}}^*=-\frac{V_{\mathbf{q}}}{\overline{C}_q}\sigma_m(\mathbf{q}). \quad (9)$$

Hence the energy functional becomes

$$E_m=\left(\frac{\hbar^2}{2\mu}\eta^2+\frac{1}{2}\mu\omega^2\frac{1}{\eta^2}\right)(|m|+1)+m\frac{\hbar\omega_c}{2} -\sum_{\mathbf{q}}\frac{|V_{\mathbf{q}}|^2}{\overline{C}_q}\,|\sigma_m(\mathbf{q})|^2, \quad (10)$$

where $\omega=(\omega_\bullet^2+\omega_c^2/4)^{1/2}$ is the strength of total confinement in the lateral plane due to both magnetic field and spatial confinement, which are characterized by the cyclotron frequency $\omega_c = eB/\mu c$ and the confinement frequency $\omega_\bullet$, respectively. By means of equation (5), $\sigma_m(\mathbf{q})$ can easily be calculated, and the result is

$$\sigma_m(\mathbf{q}) = {}_1F_1\left(|m|+1, 1; -\frac{(1-\lambda)^2 q^2}{4\eta^2}\right), \qquad (11)$$

where ${}_1F_1$ is the hypergeometric function. For convenience, in equation (10), we introduce the dimensionless parameters $(\hbar/2\mu\omega_0)^{1/2}\eta = 1/\overline{\eta}$ and $(\hbar/2\mu\omega_0)^{1/2} q = x$, and make E_m dimensionless dividing by $\hbar\omega_0$. After inserting equations (2) and (11) into equation (10), and then converting the sum in equation (10) into an integral over the introduced dimensionless variables result in

$$\overline{E}_m = \left(\frac{1}{\overline{\eta}^2} + \frac{1}{4}\overline{\omega}^2\eta^2\right)(|m|+1) + m\frac{\overline{\omega}_c}{2} - \alpha\int_0^\infty \frac{\mathrm{d}x}{1+\lambda^2x^2}\mathrm{e}^{-(1-\lambda)^2x^2\overline{\eta}^2/2} \times \left[L_{|m|}\left(\frac{(1-\lambda)^2x^2\overline{\eta}^2}{4}\right)\right]^2 \qquad (12)$$

where $L_{|m|}(x)$ are Laguerre polynomials, and $\overline{\omega} = \omega/\omega_0$, $\overline{\omega}_c = \omega_c/\omega_0$. Here, we have used the following relation [18]

$$ {}_1F_1\left(|m|+1, 1; -x^2\right) = \mathrm{e}^{-x^2}{}_1F_1\left(-|m|, 1; x^2\right) = \mathrm{e}^{-x^2}L_{|m|}\left(x^2\right). \qquad (13)$$

Equation (12) is our fundamental result, from which we obtain the ground($m = 0$) and first ($m = \pm 1$) excited states by minimization with respect to λ and $\overline{\eta}$.

Another interesting quantity is the potential due to the polarization cloud, which is defined by

$$\Phi(\mathbf{r}) = \langle\Psi|\,\varphi\left(r-r'\right)|\Psi\rangle$$

where

$$\varphi(r) = -\frac{1}{e}\sum_{\mathbf{q}}\left(V_{\mathbf{q}}b_{\mathbf{q}}\mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}} + \mathrm{h.c.}\right).$$

Within the present approach it can be shown that the polarization potential becomes

$$\Phi_m(\mathbf{r}) = -\frac{2}{e}\hbar\omega_0 \times \int_0^\infty \frac{\mathrm{d}y}{1+\lambda^2y^2}\,{}_1F_1\left(|m|+1, 1; -\frac{(1-\lambda)^2y^2\overline{\eta}^2}{4}\right)J_0(y\overline{r}) \qquad (14)$$

where $\overline{r} = r/r_0$, $y = r_0 q$ and $J_0(y\overline{r})$ is the Bessel function.

3 Limiting cases

Before discussing the numerical solutions to equation (12), we will derive some useful analytical results for the energy levels of a magnetopolaron in 2D parabolic QD. Choosing λ equal to 0 or 1 corresponds to two different physical regimes as pointed out in the previous section. The first is defined by the choice of $\lambda = 1$ and is called extended state limit, whereas $\lambda = 0$ is completely equivalent to the Landau-Pekar method.

3.1 Extended state limit

This case is realized when the confinement length is large and the electron-phonon interaction is weak, therefore the electron wave function is spread over many lattice sites. In this limit, equation (12) becomes

$$\overline{E}_m = \left(\frac{1}{\overline{\eta}^2} + \frac{1}{4}\overline{\omega}^2\overline{\eta}^2\right)(|m|+1) + m\frac{\overline{\omega}_c}{2} - \alpha\frac{\pi}{2}\cdot \qquad (15)$$

Minimization with respect to $\overline{\eta}$ gives $\overline{\eta}^2 = 2/\overline{\omega}$, thus the energy results in $\overline{E}_m = (|m|+1)\,\overline{\omega} + (m\overline{\omega}_c - \alpha\pi)/2$. This result is reduced to $\overline{E}_0\,|_{\overline{\omega}_c=0} = \overline{\omega} - \alpha\pi/2$ of the result of MC for the ground-state energy in the absence of a magnetic field. As seen from equation (15), since the interaction part of the energy is independent of the quantum number m and the total confinement frequency $\overline{\omega}$, the polaronic correction $\Delta E_m = -\left[\overline{E}_m - \overline{E}_m(\alpha = 0)\right]$ is same for all levels and is equal to $\alpha\pi/2$.

In this limit the polarization potential takes the form

$$\Phi_m(\mathbf{r}) = -\frac{2}{e}\hbar\omega_0\int_0^\infty \frac{\mathrm{d}y}{1+y^2}J_0(y\overline{r}) = -\frac{2}{e}\hbar\omega_0\frac{\pi}{2}\left[I_0(\overline{r}) - L_0(\overline{r})\right] \qquad (16)$$

where $I_0(\overline{r})$, $L_0(\overline{r})$ are the Struve functions [19]. As expected, the polarization potential is also independent of the quantum number m and the total confinement frequency $\overline{\omega}$.

3.2 Localized state limit

When we take $\lambda = 0$ and introduce the new variable $x\overline{\eta}/2 = y$ in equation (12), we obtain

$$\overline{E}_m = \left(\frac{1}{\overline{\eta}^2} + \frac{1}{4}\overline{\omega}^2\overline{\eta}^2\right)(|m|+1) + m\frac{\overline{\omega}_c}{2} - \frac{2\alpha}{\overline{\eta}} \times \int_0^\infty \mathrm{d}y\mathrm{e}^{-2y^2}\left[L_{|m|}\left(y^2\right)\right]^2. \qquad (17)$$

Minimization of $\overline{E}_m$ of equation (17) with respect to $\overline{\eta}$ gives a fourth order algebraic equation for $\overline{\eta}$ in the form

$$\frac{1}{2}\overline{\omega}^2\overline{\eta}^4 + \frac{2\alpha}{|m|+1}\overline{\eta}\int_0^\infty \mathrm{d}y\mathrm{e}^{-2y^2}\left[L_{|m|}\left(y^2\right)\right]^2 - 2 = 0, \qquad (18)$$

which is to be solved numerically after the calculation of the relevant integral for a certain value of m.

If there is a strong confinement (or strong magnetic field) but weak electron-phonon interaction, then from equation (18) we get $\overline{\eta}^2 = 2/\overline{\omega}$, which gives rise to the energy

$$\overline{E}_m = (|m|+1)\overline{\omega} + m\frac{\overline{\omega}_c}{2} - 2\alpha\sqrt{\frac{\overline{\omega}}{2}}\int_0^\infty \mathrm{d}y\mathrm{e}^{-2y^2}\left[L_{|m|}\left(y^2\right)\right]^2. \qquad (19)$$

This expression is reduced to the known results: for the ground-state energy and in the absence of a magnetic field it gives $\overline{E}_0 \mid_{\overline{\omega}_c=0}= \overline{\omega}_\bullet - \alpha\sqrt{\pi\overline{\omega}_\bullet}/2$, which is the result of the path integral method [9], whereas if we remove the confining potential it gives $\overline{E}_0 \mid_{\overline{\omega}_\bullet=0}= \left(\overline{\omega}_c - \alpha\sqrt{\pi\overline{\omega}_c/2}\right)/2$, which is the lowest Landau level [17].

On the other hand, if there is a strong electron-phonon interaction and weak confinement (and/or weak magnetic field), then from equation (18) we get $\overline{\eta} = (|m|+1)/\alpha K_m$, which gives rise to the energy

$$\overline{E}_m = \frac{1}{4}\overline{\omega}^2 \frac{(|m|+1)^3}{\alpha^2 K_m^2} + m\frac{\overline{\omega}_c}{2} - \frac{\alpha^2 K_m}{(|m|+1)} \tag{20}$$

where $K_m = \int_0^\infty \mathrm{d}y \mathrm{e}^{-2y^2}\left[L_{|m|}\left(y^2\right)\right]^2$. This expression is reduced to the ground state energy of MC, in the absence of a magnetic field

$$\overline{E}_0 \mid_{\overline{\omega}_c=0}= -\frac{\pi}{8}\alpha^2 + \frac{2}{\pi}\frac{\overline{\omega}_\bullet^2}{\alpha^2}\cdot \tag{21}$$

In this limit, the polarization potential can be easily calculated and found as

$$\Phi_0(\mathbf{r}) \mid_{\lambda=0}= -\alpha\frac{\hbar\omega_0}{e}\frac{\sqrt{2\pi}}{\gamma}\; {}_1F_1\left(\frac{1}{2}, 1; -\frac{\overline{r}^2}{2\gamma^2}\right) \tag{22}$$

for the ground state.

4 The ground state

When we take $m = 0$ in equation (12) and arrange the remaining expression, we obtain

$$\overline{E}_0 = \frac{1}{\overline{\eta}^2} + \frac{1}{4}\overline{\omega}^2\overline{\eta}^2 - \frac{\pi\alpha}{2\lambda}\left[1 - \operatorname{erf}\left(\frac{(1-\lambda)\overline{\eta}}{\sqrt{2}\lambda}\right)\right]\mathrm{e}^{(1-\lambda)^2\overline{\eta}^2/2\lambda^2} \tag{23}$$

where erf is the error function. For specified values of α, $\overline{\omega}_c$ and $\overline{\omega}_\bullet$ (or the dimensionless confinement length $u_\bullet = \sqrt{2/\overline{\omega}_\bullet}$), this equation is minimized numerically with respect to $\overline{\eta}$ and λ, and then the polaronic correction $\Delta E_0 = -(\overline{E}_0 - \overline{\omega})$ for the ground state is calculated.

In Figure 1, we plot ΔE_0, as a function of α, where the thin straight line corresponds to the extended state limit. This correction is obtained from equation (12) for $\lambda = 1$ and $m = 0$, which is equal to $\Delta E_0 = \alpha\pi/2$ and exact, and also independent of $\overline{\omega}$. In the other limit, $\lambda = 0$, when there is a strong confinement ($u_\bullet = 1$) and a magnetic field ($\overline{\omega}_c = 1$), the polaronic correction calculated from equation (23) is shown by the dashed curve in Figure 1. It appears that this curve lies below the thin line for the extended state up to $\alpha = 2$, beyond which the curve overtakes the line. Therefore this limiting case is to be valid above $\alpha = 2$. If we remove the confinement and the magnetic field, and if we calculate optimized results then we obtain the solid curve in Figure 1. This lies below the extended state up to about $\alpha = 3.8$; beyond this value the localized state exceeds the extended one. If we now calculate the polaronic correction for the optimum value of λ from equation (23), and plot the polaronic correction for $u_\bullet = 1$ and $\overline{\omega}_c = 1$, then we obtain the dotted curve in Figure 1. This curve lies above all the limiting cases and is valid for all coupling strength.

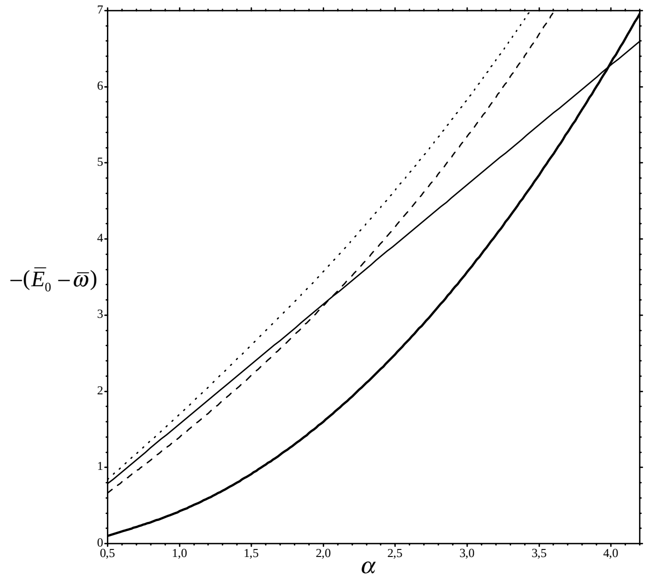

Fig. 1. The polaronic correction for the ground-state energy, equation (12), as a function of α, in comparison with three different values of the variational parameter λ. The thick and dashed lines refer to $\lambda = 0$ (strong coupling limit), for $u_\bullet = \infty$ and $\overline{\omega}_c = 0$ (bulk limit) and $u_\bullet = 1$, $\overline{\omega}_c = 1$, respectively, while the thin (straight) line represents $\lambda = 1$ (extended state limit). All coupling regime ($0 < \lambda < 1$) is represented by the dotted line for $u_\bullet = 1$ and $\overline{\omega}_c = 1$.

In Figure 2, ΔE_0 is plotted with respect to the confinement length $u_\bullet$ for $\alpha = 2$ and 4 and for various magnetic fields. Since the increasing magnetic field affects in the same manner as the decreasing confinement length, the stronger magnetic field leads to the more confined polaron. As seen in Figure 2, as decreasing confinement length, ΔE_0 increases deeply for each value of α and curves for various magnetic fields join asymptotically together. At the other side, each curve goes to the bulk polaron limit when the confinement length increases.

The polarization potential $\overline{\Phi}_0 = e\Phi_0/\hbar\omega_0$ is plotted as a function of r/r_0 in Figure 3, where the solid curve is obtained from equation (16) and represent extended states ($\lambda = 1$). It should be noted that this limiting case is independent of the confinement length and the magnetic field. On the other hand the horizontal dotted line is plotted from equation (22) for $\alpha = 2$, $\overline{\omega}_c = 1$ and without any confinement, and represents localized states. It appears that this curve is slightly dependent on r/r_0. The optimum values of the potential lie between these two limiting cases; in Figure 3, the solid lines correspond to the potential with a confinement $u_\bullet = 1$ and without any confinement $u_\bullet = \infty$ in a magnetic field, whereas the dashed curves correspond to the potential in the absence of a magnetic field. The magnetic field lowers the potential in the small values of r/r_0.

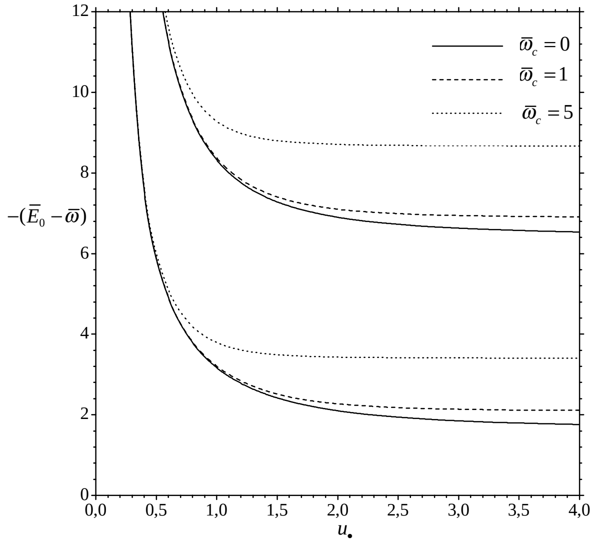

Fig. 2. The polaronic correction as a function of the confinement length for various values of magnetic field. The upper curves are for $\alpha = 4$ while the lowers are for $\alpha = 2$.

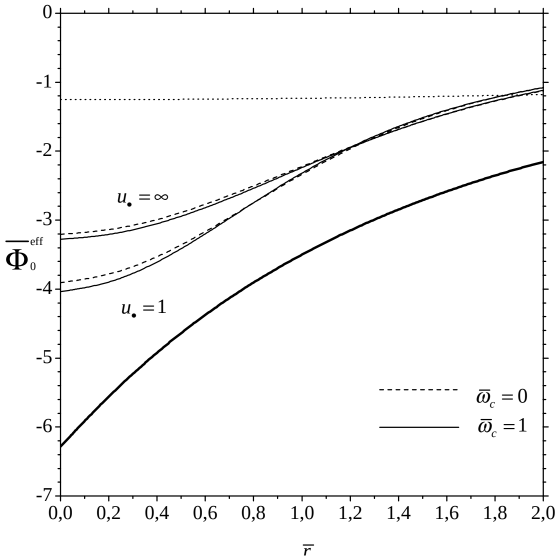

Fig. 3. The polarization potential $\overline{\Phi}_0 = e\Phi_0/\hbar\omega_0$ for the ground-state as a function of $\overline{r} = r/r_0$. For comparison, both the extended and localized state limits represented, respectively by thick line and dotted lines are given for $u_\bullet = \infty$, $\overline{\omega}_c = 1$. The curves between these two belong to all coupling regime for different values of both magnetic field and confinement length as indicated in the figure.

5 The first excited states

For the first-excited states if we take $m = \pm 1$ in equation (12), then after some tedious calculations we obtain

$$\overline{E}_{\pm 1} = 2\left(\frac{1}{\overline{\eta}^2} + \frac{1}{4}\overline{\omega}^2\overline{\eta}^2\right) \pm \frac{\overline{\omega}_c}{2} - \frac{\alpha\pi}{2}\left(\frac{\overline{\eta} + \sqrt{2}t}{\overline{\eta}}\right) \times \left[-\frac{t}{8\sqrt{\pi}}\left(7 + 2t^2\right) + \left(1 + t + \frac{t^4}{4}\right)\left(1 - \operatorname{erf}(t)\right) \mathrm{e}^{t^2}\right] \tag{24}$$

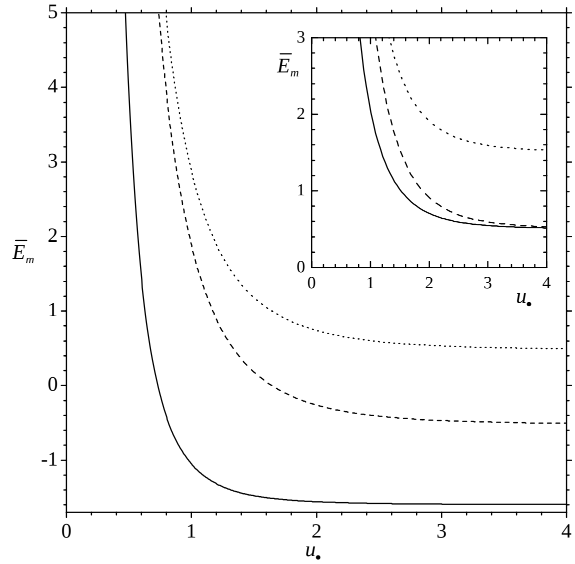

Fig. 4. The ground (solid)- and first-excited state energies with $m = -1$ (dashed) and $m = +1$ (dotted) of 2D magnetopolaron as a function of confinement length for $\alpha = 2$. Inset shows the unperturbed ($\alpha = 0$) levels.

where $t = (1 - \lambda)\overline{\eta}/\sqrt{2}\lambda$ is a new parameter used in place of λ. This equation is now minimized with respect to $\overline{\eta}$ and t.

The change of the energy $\overline{E}_m$ as a function of the confinement length $u_\bullet$ is plotted in Figure 4 for $\alpha = 2$ and $\overline{\omega}_c = 1$, where the inset indicates the same dependence in the absence of the electron-phonon interaction. The ground and first excited states go to the bulk polaron limit as the confinement length increases, whereas when $\alpha = 0$, $\overline{E}_{(-1)}$ and $\overline{E}_{(0)}$ levels join together as $u_\bullet$ increases and the degeneracy is restored. Furthermore, the separation of the energy levels are more substantial compared with those of the results of MC, which can be ascribed to the presence of a magnetic field.

The renormalized cyclotron mass ($\overline{m}^*_{\mp m}$) which is the ratio of cyclotron mass ($m_{\mp m}$) to the electron band mass μ is defined to be $\overline{m}^*_{\mp m} = \overline{\omega}_c/[\overline{E}_{\mp(m+1)} - \overline{E}_{\mp m}]$ which depends on the relevant cyclotron resonance frequency for the allowed transitions $\mp(m+1) \longrightarrow m$. As done for the magnetopolaron energy levels, one can obtain approximate analytical expressions for the renormalized cyclotron mass both for extended and localized limits. Figure 5 shows the renormalized cyclotron mass obtained numerically associated with the transitions $\mp 1 \longrightarrow 0$ as a function of confinement length $u_\bullet$ for a fixed value of magnetic field, *i.e.* $\overline{\omega}_c = 1$. From the figure, it can be clearly seen that the renormalized cyclotron masses decrease with increasing both confinement length and coupling strength.

6 Conclusion

In this paper, we have performed a variational calculation for all coupling strength to obtain the ground and first excited state energies of a magnetopolaron in

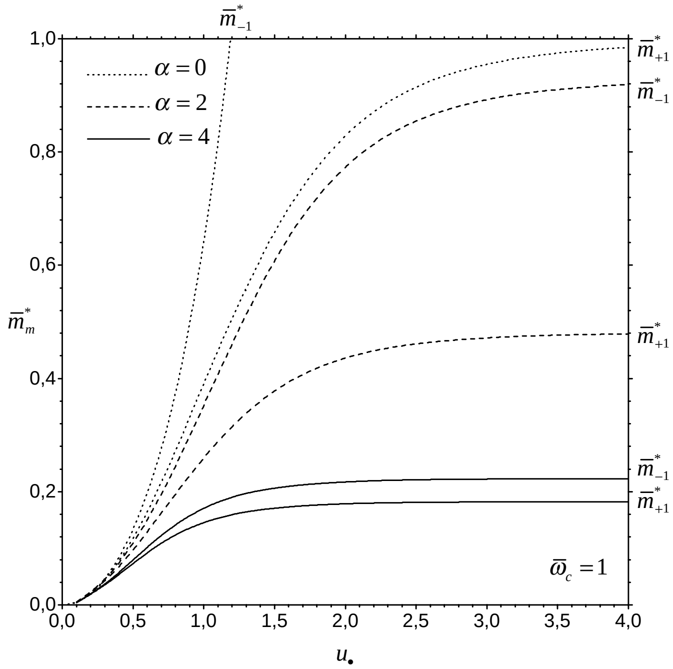

Fig. 5. The cyclotron masses associated with the transition ($\mp 1 \rightarrow 0$) as a function of confinement length at fixed magnetic field $\overline{\omega}_c = 1$ for various electron-phonon coupling strength.

a parabolic QD. The variational technique that is developed by MC within the approach by the LLPH method has been extended to a polaron in a magnetic field. The ground and first excited state energies have been obtained and removal of degenerecies by the confinement and the electron-phonon interaction has been examined for all coupling strength and for various limiting cases. The polarization potential and its dependence on the magnetic field have also been examined in the same framework. Our present investigations shows that presence of spatial confinement in addition to electron-phonon interaction leads to noticeable effects in the behavior of the cyclotron mass of a magnetopolaron confined in a 2D parabolic QD under the variation of confinement length and electron-phonon coupling strength.

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References

1. N.F. Johnson, J. Phys. Cond. Matt. **7**, 965 (1995); T. Demel, D. Heitmann, P. Grambow, K. Ploog, Phys. Rev. Lett. **64**, 788 (1990); A. Lorke, J.P. Kotthaus, K. Ploog, *ibid.* **64**, 2559 (1990)
2. J.S. Pan, H. Pan, Phys. Status Solidi (b) **148**, 129 (1988); S.N. Klimin, E.P. Pokatilov, V.M. Fomin, Phys. Status Solidi (b) **184**, 373 (1994); K. Oshiro, K. Akai, M. Matsuara, Phys. Rev. B **58**, 7986 (1998); S. Sahoo, Phys. Lett. A **238**, 390 (1998); S. Sahoo, Z. Phys. B **101**, 97 (1996)
3. M.H. Degani, G.A. Farias, Phys. Rev. B **42**, 11950 (1990); S. Mukhopadhyay, A. Chatterjee, Phys. Lett. A **204**, 411 (1995); Y. Lépine, G. Bruneau, J. Phys. Cond. Matt. **10**, 1495 (1998); K.D. Zhu, S.W. Gu, Phys. Lett. A **163**, 435 (1992); S. Mukhopadhyay, A. Chatterjee, Phys. Lett. A **240**, 100 (1998); S. Mukhopadhyay, A. Chatterjee, *ibid.* E **240**, 100 (1998); S. Mukhopadhyay, A. Chatterjee, Phys. Rev. B **58**, 2088 (1998)
4. K.D. Zhu, S.W. Gu, Phys. Rev. B **47**, 12941 (1993); K.D. Zhu, T. Kobayashi, Phys. Lett. A **190**, 337 (1994); Q.P. Lie, K. Karrai, S.K. Yip, S. Das Sarma, H.D. Drew, Phys. Rev. B **43**, 5151 (1991)
5. J.T. Devreese, *Encyclopedia of Applied Physics* (VCH Publishers, Weinheim, 1996), Vol. 14, pp. 383–413
6. J.T. Devreese, F. Brosens, Phys. Rev. B **45**, 6459 (1992)
7. W.J. Huybrechts, J. Phys. C Solid State Phys. **10**, 3761 (1977)
8. S. Mukhopadhyay, A. Chatterjee, J. Phys. Cond. Matt. **11**, 2071 (1999)
9. S. Mukhopadhyay, A. Chatterjee, Int. J. Mod. Phys. B **10**, 2781 (1996)
10. Ch. Sikorski, U. Merkt, Phys. Rev. Lett. **62**, 2164 (1989); B. Meurer, D. Heitmann, K. Ploog, *ibid.* **68**, 1371 (1992)
11. M. Nirmal, C.B. Murray, D.J. Norris, M.G. Bawendi, Z. Phys. D **26**, 361 (1993); V. Jungnickel, F. Henneberger, J. Lumin. **70**, 238 (1996)
12. M.C. Klein, F. Hache, D. Ricard, C. Flytzanis, Phys. Rev. B **42**, 11 123 (1990)
13. G. Lindemann, R. Lassnig, W. Seidenbusch, E. Gornik, Phys. Rev. B **28**, 4693 (1983)
14. S. Mukhopadhyay, F.M. Peetrs, J. Phys. Cond. Matt. **14**, 8005 (2002)
15. F.M. Peeters, J.T. Devreese, Phys. Status Solidi (b) **115**, 285 (1983)
16. F.M. Peeters, WU Xiaoguang, J.T. Devreese, Phys. Rev. B **33**, 3926 (1986)
17. B.S. Kandemir, T. Altanhan, Il Nuovo Cimento D **20** (1998)
18. M. Abramowitz, I.A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1972), pp. 509
19. I.S. Gradshteyn, I.M. Ryzhik, *Tables of Integrals, Series and Products* (Academic, New York, 1979), pp. 678
20. E. Pokatilov, S.N. Klimin, S.N. Balaban, V.M. Fomin, Phys. Status Solidi (b) **189**, 433 (1995)